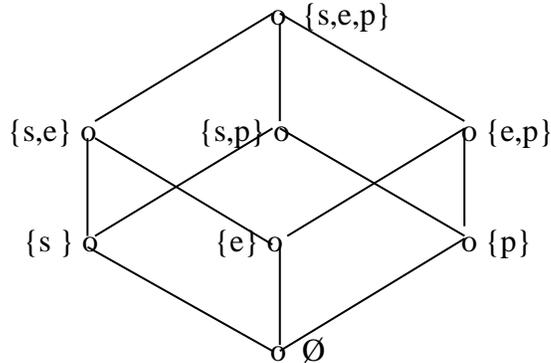


Boolean semantics for plurality

Let $\{s, e, p\}$ be the set containing Sasha, Emma and Pim. Look at $\text{pow}(\{s, e, p\})$:
 $\{\emptyset, \{s\}, \{e\}, \{p\}, \{s, e\}, \{s, p\}, \{e, p\}, \{s, e, p\}\}$. We think of this set as ordered by $\subseteq, \cap, \cup, -$

D

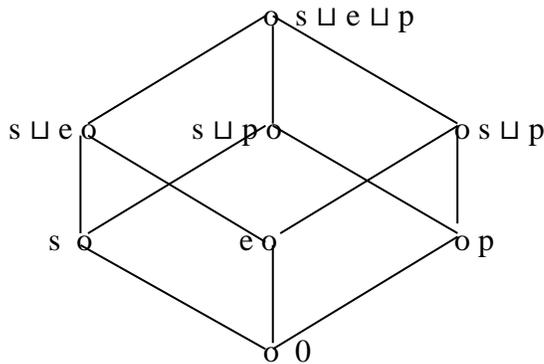


\subseteq **subset** $\emptyset \subseteq \{s\} \subseteq \{s, e\}$
 \cup **union** $\{s\} \cup \{e\} = \{s, e\}$
 \cap **intersection** $\{s, e\} \cap \{s, p\} = \{s\}$
 $-$ **complement** $D - \{s\} = \{e, p\}$

This structure is called a Boolean algebra.

We can impose **the same structure** on the domain of individuals, i.e. ignoring the set nature of the objects in the above structure:

D



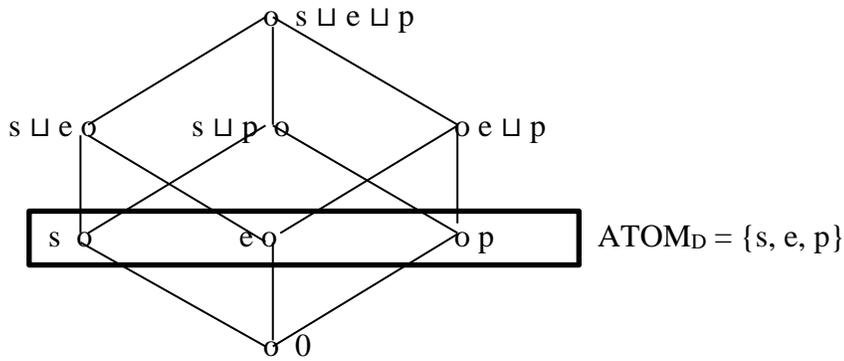
\sqsubseteq **part-of** $0 \sqsubseteq s \sqsubseteq s \sqcup e$
 \sqcup **sum** $\sqcup \{s, e\} = s \sqcup e$
 \sqcap **overlap** $\sqcap \{s \sqcup e, s \sqcup p\} = s$
 $-$ **remainder** $\neg s = e \sqcup p$

$s \sqsubseteq s \sqcup e$, s is part of the sum of s and e
the overlap of $s \sqcup e$ and $s \sqcup p$ is s
take away s from $s \sqcup e \sqcup p$: $(\neg s)$
you are left with $e \sqcup p$

This is a Boolean algebra of singular and plural objects.

1. 0 is the null entity.
2. D^+ , the set of **objects**, is $D - \{0\}$
3. Let $d_1, d_2 \in D^+$: d_1 and d_2 **overlap** iff $d_1 \sqcap d_2 \in D^+$
 d_1 and d_2 are **disjoint** iff $d_1 \sqcap d_2 = 0$
4. ATOM_D , the set of **atoms** in D is the set of **minimal objects** in D^+ :
 d_1 is **minimal** in D^+ iff for every $d_2 \in D^+$: if $d_2 \sqsubseteq d_1$ then $d_2 = d_1$.

D



5: $ATOM_D$ is the set of singular individuals in D

6: Singular nouns denote sets of singular individuals:

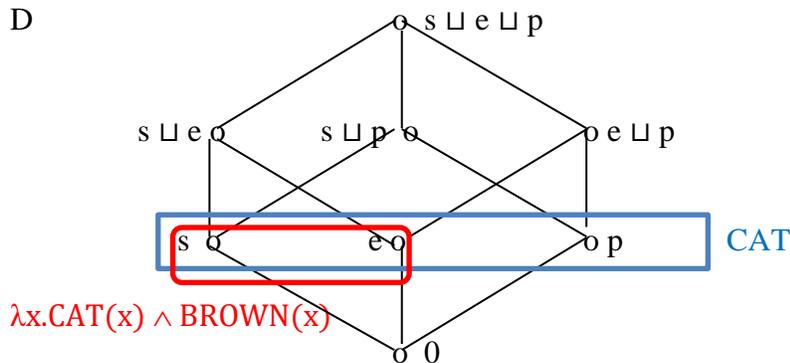
Let $CAT, BROWN \in PRED^1$

$cat \rightarrow CAT \quad F_M(CAT) \subseteq ATOM_D \quad \text{say: } F_M(CAT) = \{s, e, p\}$
 $brown \rightarrow BROWN \quad \text{say: } F_M(BROWN) = \{s, e, f\}$
 (Fido is not shown in the picture)
 $brown\ cat \rightarrow \lambda x. CAT(x) \wedge BROWN(x) \in PRED^1$

Then: $\llbracket \lambda x. CAT(x) \wedge BROWN(x) \rrbracket_{M,g} = \{s, e\} \subseteq ATOM_D$

So the complex NP *brown cat* also denotes a set of atoms, singular individuals.

D



7. Atoms and singularity: $ATOM_D$: the set of singular individuals

$D^+ - ATOM_D$: the set of plural individuals, sums of singular individuals

8 Semantic pluralization (Link 1983): semantic pluralization is **closure under sum**.

If $P \in PRED^1$ then $*P \in PRED^1$

$\llbracket *P \rrbracket_{M,g} = \{d \in D_M: \text{for some } X \subseteq \llbracket P \rrbracket_{M,g}: d = \sqcup(X)\}$

You add to the denotation of P all sums of elements of P, technically: the sum of every subset of the denotation of P (the formulation in terms of subsets is important).

Default: lexically singular nouns denote sets of atoms:
lexical pluralization is semantic pluralization

$cat \rightarrow CAT \subseteq ATOM_D$
 $cats \rightarrow *CAT$

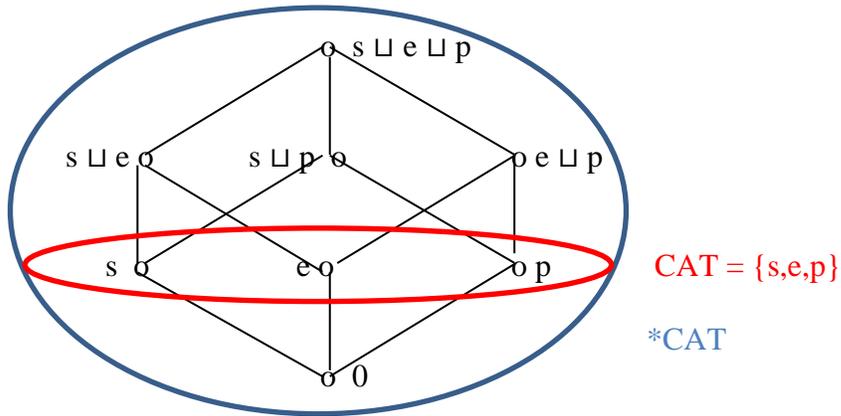
$[[CAT]]_{M,g} = \{s, e, p\}$

cat

$[[*CAT]]_{M,g} = \{0, s, e, p, s \sqcup e, s \sqcup p, e \sqcup p, s \sqcup e \sqcup p\}$

$cats$

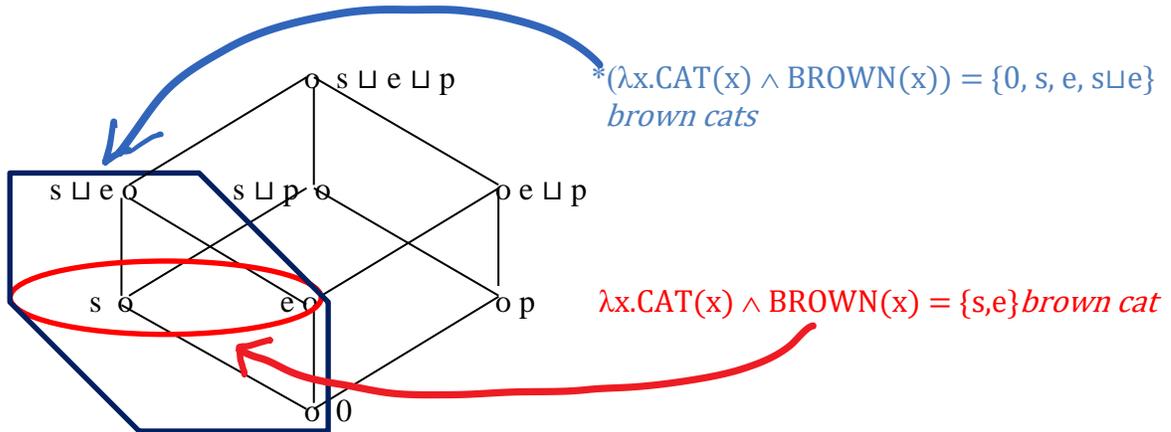
D



$[[\lambda x.CAT(x) \wedge BROWN(x)]]_{M,g} = \{s, e\}$

$[[*(\lambda x.CAT(x) \wedge BROWN(x))]]_{M,g} = \{0, s, e, s \sqcup e\}$

D



9. and as sum : sum conjunction

if $\alpha, \beta \in \text{TERM}$ then $\alpha \sqcup \beta \in \text{TERM}$

$$\llbracket \alpha \sqcup \beta \rrbracket_{M,g} = \llbracket \alpha \rrbracket_{M,g} \sqcup \llbracket \beta \rrbracket_{M,g}$$

Hence:

Sasha and Emma and Pim $\rightarrow s \sqcup e \sqcup p \in \text{TERM}$

$$\llbracket s \sqcup e \sqcup p \rrbracket_{M,g} = F_M(s) \sqcup F_M(e) \sqcup F_M(p) = s \sqcup e \sqcup p$$

are cats $\rightarrow *CAT$

Sasha and Emma and Pim are cats $\rightarrow *CAT(s \sqcup e \sqcup p)$

Lemma: $*CAT(s \sqcup e \sqcup p) \Leftrightarrow CAT(s) \wedge CAT(e) \wedge CAT(p)$

One side follows from the definition of *, the other side from the fact that CAT denotes a set of atoms and that D is a Boolean algebra.

10. Atomic parts and cardinality

Let $d \in D$.

ATOM_d , the set of **atomic parts of d** is:

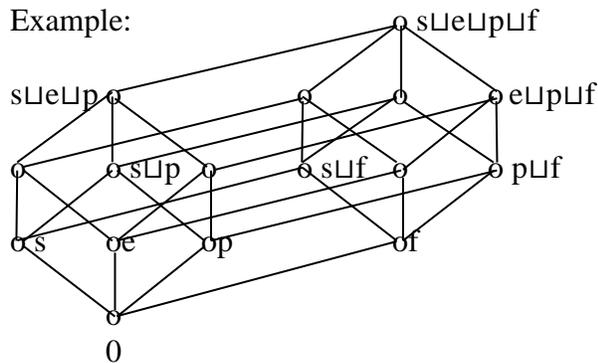
$$\text{ATOM}_d = \{a \in \text{ATOM}_D : a \sqsubseteq d\}$$

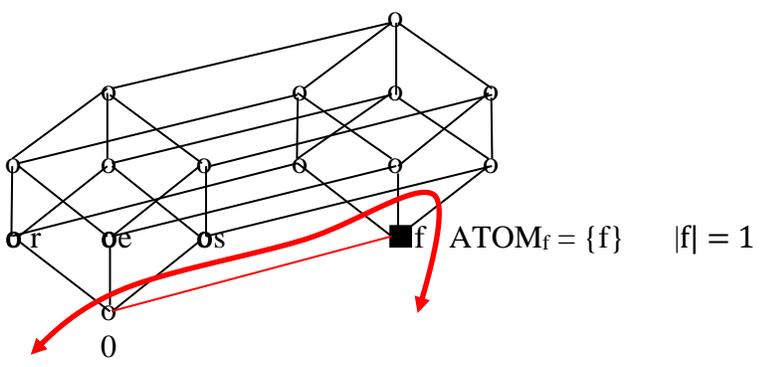
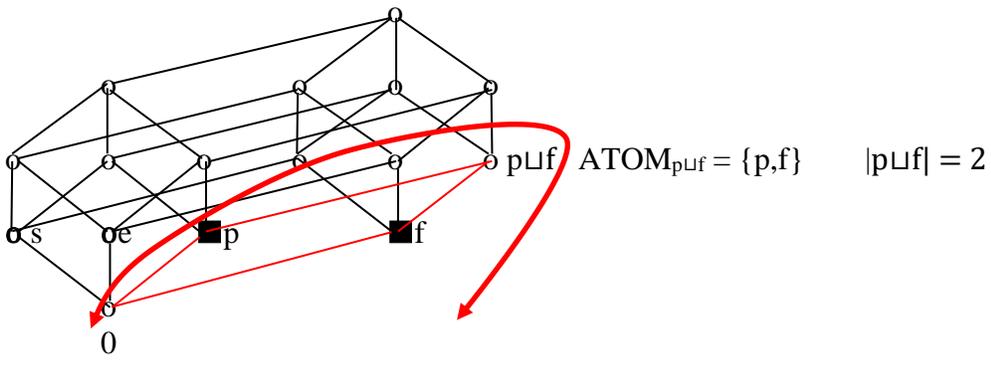
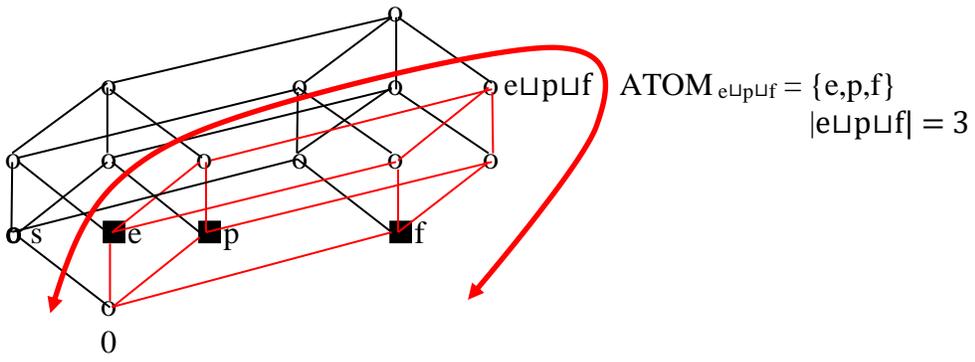
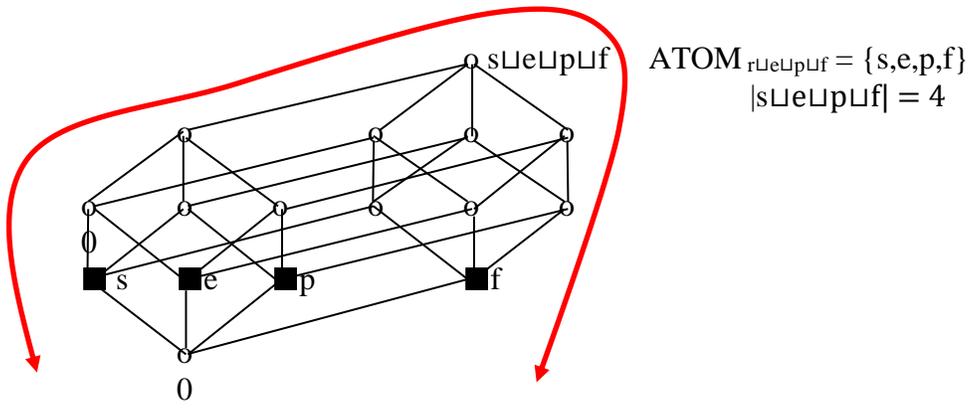
$|d|$, the cardinality of d is:

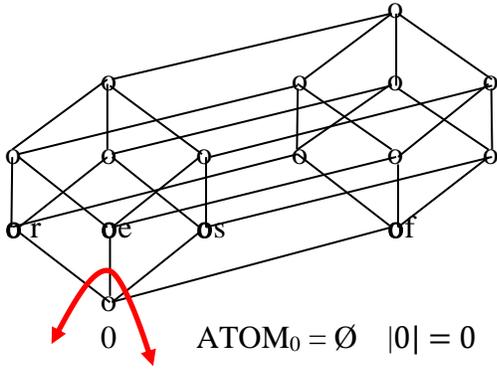
$$|d| = |\text{ATOM}_d|$$

If we have a set of atoms of four individuals $\{s,e,p,f\}$, the Boolean algebra has 16 elements:

Example:







11. Numerical adjectives

exactly two $\rightarrow \lambda x. |x| = 2$ The set of entities in D that have exactly two atomic parts

exactly two cats $\rightarrow \lambda x. *CAT(x) \wedge |x| = 2$ The set of sums of cats that have exactly two atomic parts

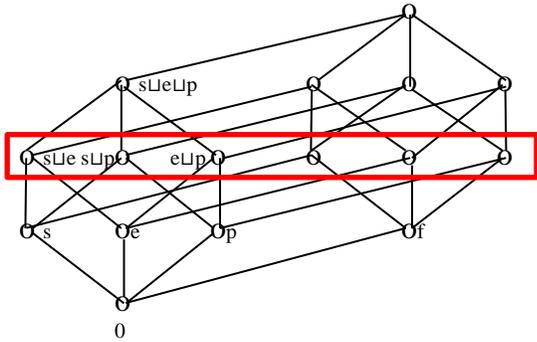
at least two $\rightarrow \lambda x. |x| \geq 2$ The set of entities in D that have at least two atomic parts

at least two cats $\rightarrow \lambda x. *CAT(x) \wedge |x| \geq 2$ The set of sums of cats that have at least two atomic parts

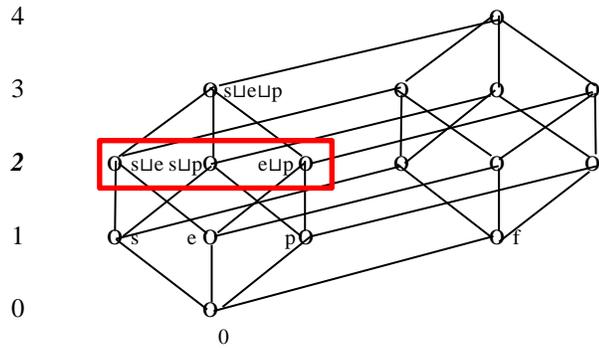
at most two $\rightarrow \lambda x. |x| \leq 2$ The set of entities in D that have at most two atomic parts

at most two cats $\rightarrow \lambda x. *CAT(x) \wedge |x| \leq 2$ The set of sums of cats that have at most two atomic parts

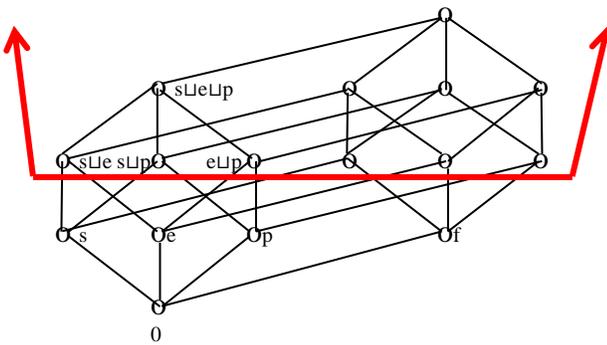
Exactly two $\rightarrow \lambda x. |x|=2$



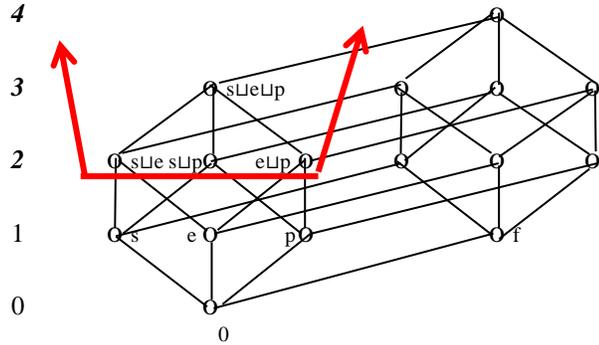
Exactly two cats $\rightarrow \lambda x. *CAT_{w,t}(x) \wedge |x|=2$



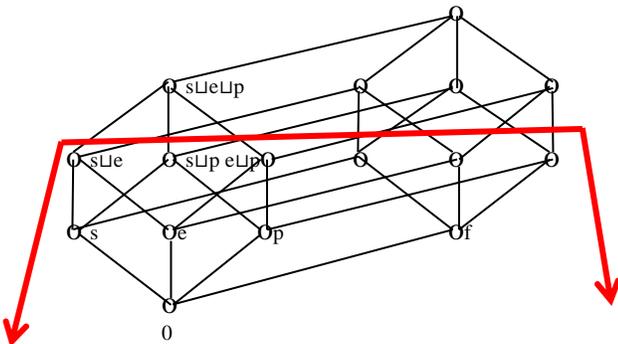
At least two $\rightarrow \lambda x. |x| \geq 2$



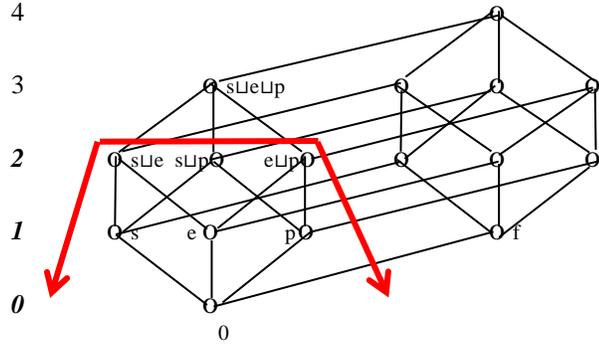
At least two cats $\rightarrow \lambda x. *CAT_{w,t}(x) \wedge |x| \geq 2$



At most two $\rightarrow \lambda x. |x| \leq 2$



At most two cats $\rightarrow \lambda x. *CAT_{w,t}(x) \wedge |x| \leq 2$



These pictures form a nice visual expression of how the polarity nature of the numerical DPs (downward entailing, upward entailing, neither up nor down) is directly determined by the number relation, \leq , \geq , $=$ on the natural numbers, i.e. \geq is closed downward on the natural numbers as indicated in the picture, \leq is closed upward, and $=$ is neither.

12. The definite article (Sharvy 1980) as a presuppositional **maximality** operation

$$\llbracket \sigma(P) \rrbracket_{M,g} = \begin{cases} \sqcup(\llbracket P \rrbracket_{M,g}) & \text{if } \sqcup(\llbracket P \rrbracket_{M,g}) \in \llbracket P \rrbracket_{M,g} \\ \perp & \text{otherwise} \end{cases}$$

$\sigma(P)$ denotes the sum of the elements in the denotation of P
if that sum is itself in the denotation of P .

$\sigma(P)$ is undefined otherwise.

This is a maximalization operation: when $\llbracket \sigma(P) \rrbracket_{M,g}$ is defined, the denotation of P , $\llbracket P \rrbracket_{M,g}$, has a maximal element $\sqcup(\llbracket P \rrbracket_{M,g})$, and $\llbracket \sigma(P) \rrbracket_{M,g}$ denotes that maximal element.

So σ is a presuppositional version of \sqcup :

$\sqcup(\llbracket P \rrbracket_{M,g})$ is defined whether or not it is in $\llbracket P \rrbracket_{M,g}$.

$\llbracket \sigma(P) \rrbracket_{M,g}$ is only defined when $\sqcup(\llbracket P \rrbracket_{M,g}) \in \llbracket P \rrbracket_{M,g}$.

the cat $\rightarrow \sigma(\text{CAT})$

the cats $\rightarrow \sigma(*\text{CAT})$

The sigma operation is a generalization of our earlier sigma operation: for **singular predicates** the new sigma does exactly what the earlier sigma did.

Case 1: Singular nouns

Let as before: $F_M(\text{CAT}) = \{s, e, p\}$

$F_M(\text{DOG}) = \{f\}$

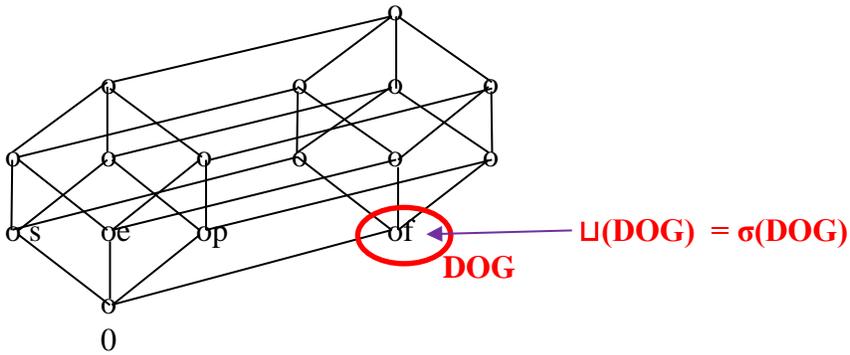
$F_M(\text{SWAN}) = \emptyset$

the dog $\rightarrow \sigma(\text{DOG})$

$\llbracket \sigma(\text{DOG}) \rrbracket_{M,g} = \sqcup(\llbracket \text{DOG} \rrbracket_{M,g}) = \sqcup(\{f\}) = f$ if $f \in \llbracket \text{DOG} \rrbracket_{M,g}$

$\llbracket \text{DOG} \rrbracket_{M,g} = \{f\}$ and $f \in \{f\}$, hence

$\llbracket \sigma(\text{DOG}) \rrbracket_{M,g} = f$



the swan $\rightarrow \sigma(\text{SWAN})$

$\llbracket \sigma(\text{SWAN}) \rrbracket_{M,g} = \sqcup(\emptyset) = 0$, if $0 \in \llbracket \text{SWAN} \rrbracket_{M,g}$

But, $\llbracket \text{SWAN} \rrbracket_{M,g} = \emptyset$ and $0 \notin \emptyset$, hence:

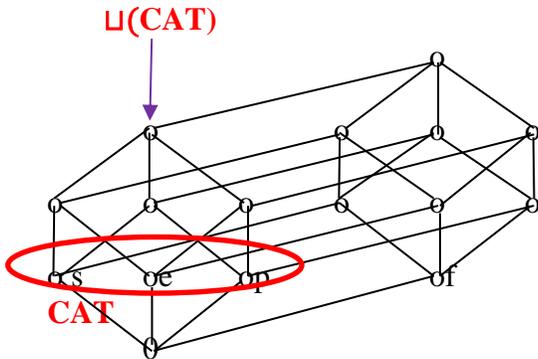
$\llbracket \sigma(\text{SWAN}) \rrbracket_{M,g} = \perp$

the cat $\rightarrow \sigma(\text{CAT})$

$\llbracket \sigma(\text{CAT}) \rrbracket_{M,g} = \sqcup(\{s, e, p\}) = s \sqcup e \sqcup p$, if $s \sqcup e \sqcup p \in \llbracket \text{CAT} \rrbracket_{M,g}$

$\llbracket \text{CAT} \rrbracket_{M,g} = \{s, e, p\}$, and $s \sqcup e \sqcup p \notin \{s, e, p\}$, hence:

$\llbracket \sigma(\text{CAT}) \rrbracket_{M,g} = \perp$

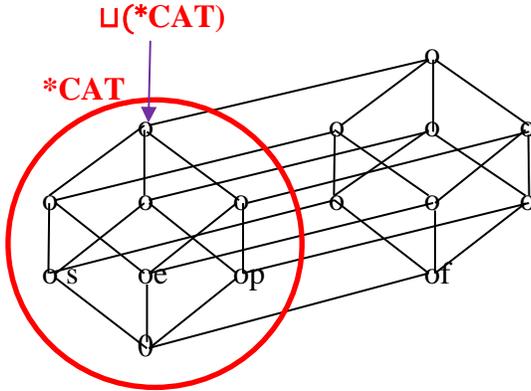


We see indeed that for singular nouns σ does what σ did before.

Case 2: Plural nouns

the cats $\rightarrow \sigma(*\text{CAT})$

$\llbracket \sigma(*\text{CAT}) \rrbracket_{M,g} = \sqcup(\{s,e,p\}) = s \sqcup e \sqcup p$, if $s \sqcup e \sqcup p \in \llbracket *\text{CAT} \rrbracket_{M,g}$
 $\llbracket *\text{CAT} \rrbracket_{M,g} = \{0, s, e, p, s \sqcup e, s \sqcup p, e \sqcup p, s \sqcup e \sqcup p\}$ and
 $s \sqcup e \sqcup p \in \{0, s, e, p, s \sqcup e, s \sqcup p, e \sqcup p, s \sqcup e \sqcup p\}$



the three cats $\rightarrow \sigma(\lambda x. *\text{CAT}(x) \wedge |x|=3)$

$\llbracket \sigma(\lambda x. *\text{CAT}(x) \wedge |x|=3) \rrbracket_{M,g} = s \sqcup e \sqcup p$

$\llbracket \lambda x. *\text{CAT}(x) \wedge |x|=3 \rrbracket_{M,g} = \{s \sqcup e \sqcup p\}$ and $s \sqcup e \sqcup p \in \{s \sqcup e \sqcup p\}$

the two cats $\rightarrow \sigma(\lambda x. *\text{CAT}(x) \wedge |x|=2)$

$\llbracket \sigma(\lambda x. *\text{CAT}(x) \wedge |x|=2) \rrbracket_{M,g} = \perp$, because

$\llbracket \lambda x. *\text{CAT}(x) \wedge |x|=2 \rrbracket_{M,g} = \{s \sqcup e, s \sqcup p, e \sqcup p\}$ and $s \sqcup e \sqcup p \notin \{s \sqcup e, s \sqcup p, e \sqcup p\}$

Case 3. Triviality and infelicity: 0 and \perp

swan \rightarrow SWAN

$$\llbracket \text{SWAN} \rrbracket_{M,g} = \emptyset$$

swans \rightarrow *SWAN

$$\llbracket \text{*SWAN} \rrbracket_{M,g} = \{0\}$$

\emptyset has exactly one subset \emptyset , and $\sqcup(\emptyset) = 0$, hence $\text{*}\emptyset = \{0\}$.

at most two swans $\rightarrow \lambda x. \text{*SWAN}(x) \wedge |x| \leq 2$

$$\llbracket \lambda x. \text{*SWAN}(x) \wedge |x| \leq 2 \rrbracket_{M,g} = \{0\}$$

This is because $|0| \leq 2$

two swans $\rightarrow \lambda x. \text{*SWAN}(x) \wedge |x|=2$

$$\llbracket \lambda x. \text{*SWAN}(x) \wedge |x|=2 \rrbracket_{M,g} = \emptyset$$

This is because $|0| \neq 2$

Hence:

the (one) swan $\rightarrow \sigma(\text{SWAN})$

$$\llbracket \sigma(\text{SWAN}) \rrbracket_{M,g} = \perp$$

because $0 \notin \emptyset$

the two swans $\rightarrow \sigma(\lambda x. \text{*SWAN}(x) \wedge |x|=2)$

$$\llbracket \sigma(\lambda x. \text{*SWAN}(x) \wedge |x|=2) \rrbracket_{M,g} = \perp$$

because $0 \notin \emptyset$

the swans $\rightarrow \sigma(\text{*SWAN})$

$$\llbracket \sigma(\text{*SWAN}) \rrbracket_{M,g} = 0$$

because $0 \in \{0\}$

the less than two swans $\rightarrow \sigma(\lambda x. \text{*SWAN}(x) \wedge |x| < 2)$

$$\llbracket \sigma(\text{*SWAN}) \rrbracket_{M,g} = 0$$

because $0 \in \{0\}$

We discussed this earlier: with the examples of the fraudulent lottery

$\llbracket \alpha \rrbracket_{M,g} = \emptyset \Rightarrow \llbracket \text{The two } \alpha \text{s} \rrbracket_{M,g} = \perp$ presupposition failure

The two persons that came with me with a lottery ticket got a prize. #Fortunately I was away.

$\llbracket \alpha \rrbracket_{M,g} = \emptyset \Rightarrow \llbracket \text{The } \alpha \text{s} \rrbracket_{M,g} = 0$ Triviality

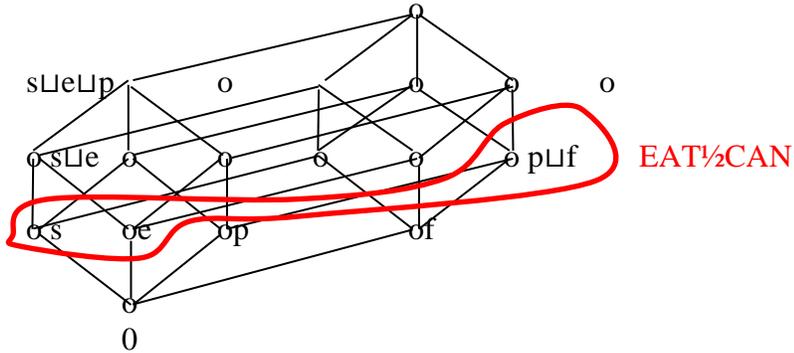
The persons that came with me with a lottery ticket got a prize. ✓Fortunately I was away.

Distributivity

Suppose Sasha and Emma eat half a can of tuna each
and Pim and Fido eat half a can of tuna together.

And Sasha and Emma are the brown cats.

Let $EAT\frac{1}{2}CAN \in PRED_1$ and $F_M(EAT\frac{1}{2}CAN) = \{s, e, p \sqcup f\}$

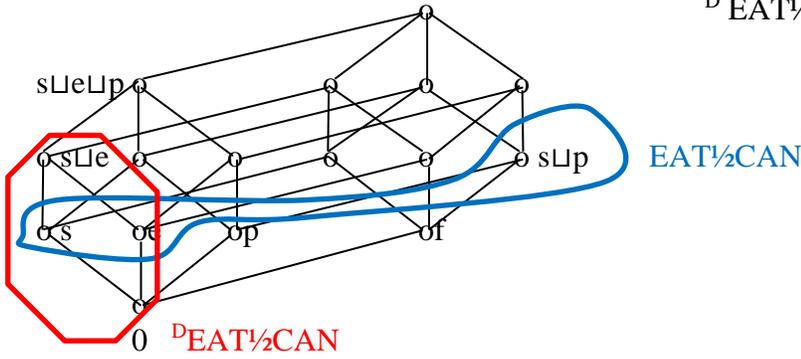


Link 1983 introduces a distributivity operator D which is used, among others for the interpretation of *each* as a VP operation,

If $P \in \text{PRED}^1$ then $D P \in \text{PRED}^1$

$\llbracket DP \rrbracket_{M,g} = \{d \in D: \text{for every } a \in \text{ATOM}_d: d \in \llbracket P \rrbracket_{M,g}\}$

$D \text{EAT}^{\frac{1}{2}}\text{CAN} = *(\text{EAT}^{\frac{1}{2}}\text{CAN} \cap \text{ATOM})$

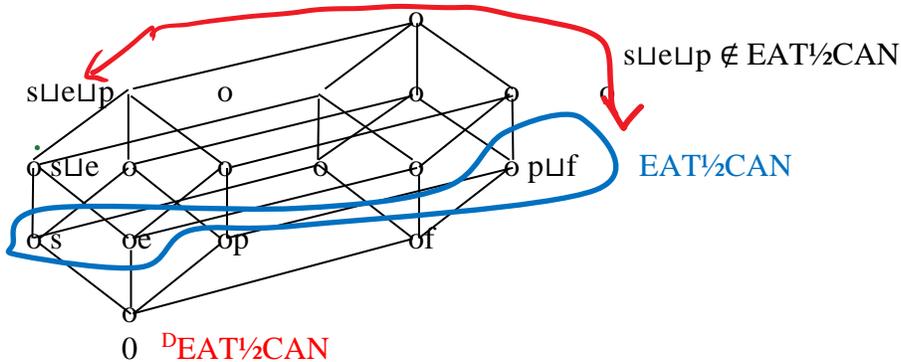


The predicate $\text{EAT}^{\frac{1}{2}}\text{CAN}$ is not itself a distributive predicate (like *are cats*), because $p \sqcup f \in \text{EAT}^{\frac{1}{2}}\text{CAN}$, but neither $p \in \text{EAT}^{\frac{1}{2}}\text{CAN}$ nor $f \in \text{EAT}^{\frac{1}{2}}\text{CAN}$.
 But $D \text{EAT}^{\frac{1}{2}}\text{CAN}$ is a distributive predicate: it takes the set of singular individuals in $\text{EAT}^{\frac{1}{2}}\text{CAN}$ and closes that set under sum.

- (1) a. The cats ate (exactly) half a can of tuna
- b. The cats [*each* ate half a can of tuna]
- c. The brown cats [*each* ate half a can of tuna]

(1a) $\rightarrow \text{EAT}^{\frac{1}{2}}\text{CAN}(\sigma(*\text{CAT}))$

This expresses that $s \sqcup e \sqcup p \in F_M(\text{EAT}^{\frac{1}{2}}\text{CAN})$, which is false, as we see.

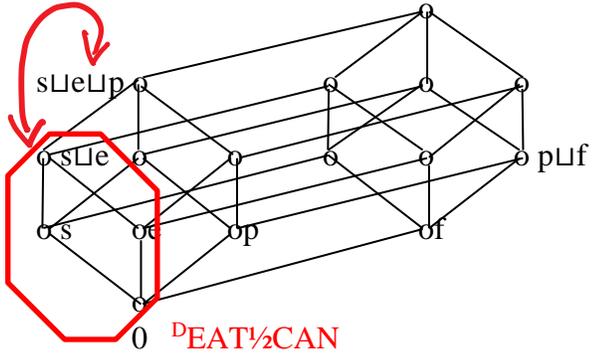


(1b) $\rightarrow \text{D}EAT\frac{1}{2}CAN(\sigma(*CAT))$

This expresses that $s\sqcup e\sqcup p \in *(\{s,e\})$, which is false, as we see,

$$\text{D}EAT\frac{1}{2}CAN = *(EAT\frac{1}{2}CAN \cap ATOM)$$

$$s\sqcup e\sqcup p \notin \text{D}EAT\frac{1}{2}CAN$$



$EAT\frac{1}{2}CAN$

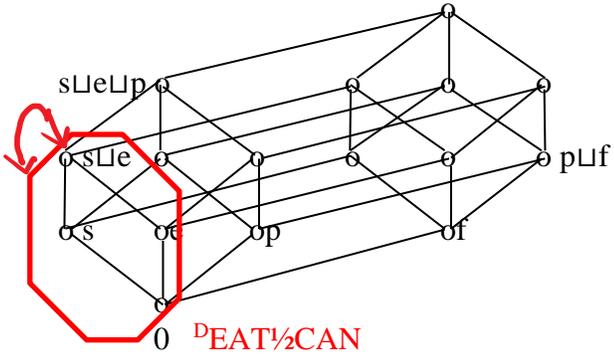
$\text{D}EAT\frac{1}{2}CAN$

(1c) $\rightarrow \text{D}EAT\frac{1}{2}CAN(\sigma(*(\lambda x.BROWN(x) \wedge CAT(x))))$

This expresses that $r\sqcup e \in *\{r,e\}$, which is true.

$$\text{D}EAT\frac{1}{2}CAN = *(EAT\frac{1}{2}CAN \cap ATOM)$$

$$s\sqcup e \in \text{D}EAT\frac{1}{2}CAN$$



$EAT\frac{1}{2}CAN$

$\text{D}EAT\frac{1}{2}CAN$

Neo-davidsonian event semantics for plurality.

1. Singular neo-Davidsonian event semantics.

I will not discuss the motivation for neo-Davidsonian event semantics here (an introduction is given in Advanced Semantics), but describe the basic set up.

	Predicate Logic:	Neo-Davidsonian Event Semantics
	D_M set of individuals	D_M and E_M set of events
<i>cat</i>	1-place predicate of individuals CAT(x)	1-place predicate of individuals CAT(x)
<i>purr</i>	PURR 1-place predicate of individuals PURR(x)	PURR 1-place predicate of events PURR(e)
<i>chase</i>	2-place relation between individuals CHASE(x,y)	CHASE 1-place predicate of events CHASE(e)

So what about x and y in event semantics?

Thematic roles are partial functions from events to individuals:

Agent: Ag: $E_M \rightarrow D_M$

The agent role specifies for events that have an agent what their agent is.

For instance, it may specify for a specific event e_1 of purring that its agent is Sasha:

Ag(e_1) = Sasha

Theme: Th: $E_M \rightarrow D_M$

Thematic roles specify event participants.

This generalizes:

The temporal trace function is a partial function τ that maps an event e and a world w onto the interval of time $\tau_w(e)$ at which event e goes on in w (if it does).

The logical language has predicates of individuals, predicates of events, roles expressions, and abstraction and quantification over individual variables and over event variables.

Correspondence:

Predicate Logic:

Sasha purr

PURR(s)

Neo-Davidsonian Event Semantics

$\exists e[\text{PURR}(e) \wedge \text{Ag}(e)=s]$

There is a purring event with Sasha as agent

Sasha chase Fido

CHASE(s, f)

$\exists e[\text{CHASE}(e) \wedge \text{Ag}(e)=s \wedge \text{Th}(e)=f]$

There is a chasing event with Sasha as agent and Fido as theme

Sasha chased Fido

$\exists e[\text{CHASE}(e) \wedge \text{Ag}(e)=s \wedge \text{Th}(e)=f \wedge \tau_w(e)<\text{now}]$

There is a chasing event with Sasha as agent and Fido as theme which is realized in w at an interval before now.

Some cat chased some dog

$\exists x[\text{CAT}(x) \wedge \exists y[\text{DOG}(y) \wedge \exists e[\text{CHASE}(e) \wedge \text{Ag}(e)=x \wedge \text{Th}(e)=y \wedge \tau_w(e)<\text{now}]]]$

There is a cat and there is a dog and a chasing event of that cat chasing that dog realized in w before now.

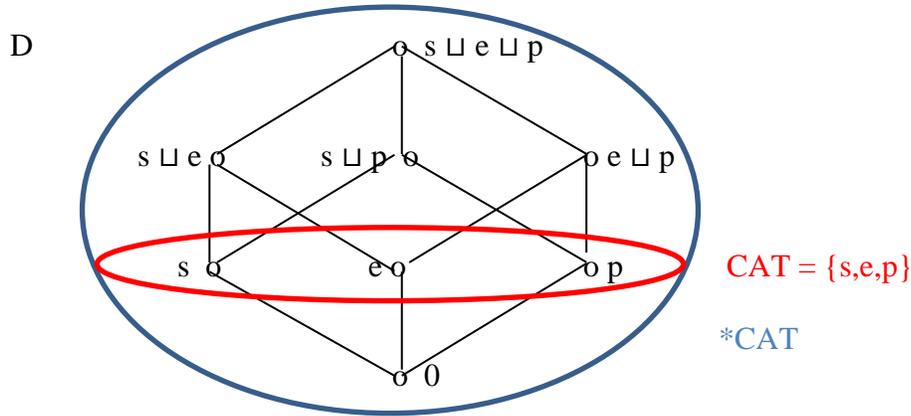
(See Landman 1980, Events and Plurality for details, or Advanced Semantics).

2. Plural Neo-Davidsonian event semantics.

In Plural Neo=Davidsonian event semantics we have pluralization of individual predicates

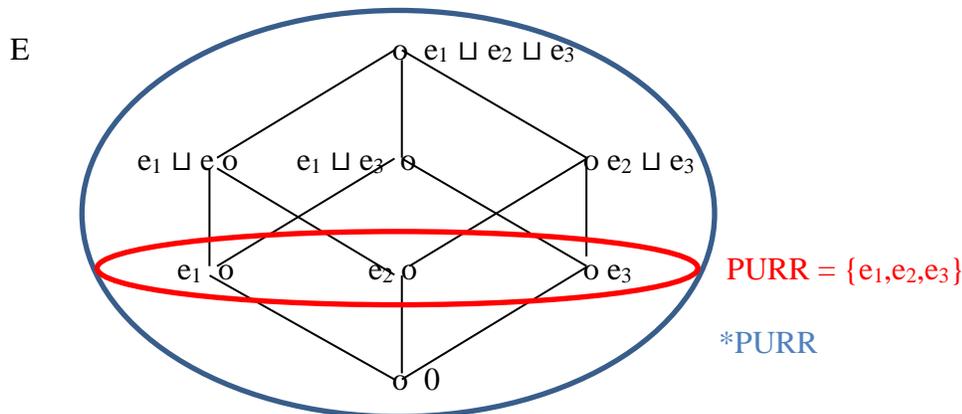
*CAT is the closure under sum of CAT

$$\begin{aligned} \llbracket \text{CAT} \rrbracket_{M,w,g} &= \{s, e, p\} && \text{cat} \\ \llbracket * \text{CAT} \rrbracket_{M,w,g} &= \{0, s, e, p, s \sqcup e, s \sqcup p, e \sqcup p, s \sqcup e \sqcup p\} && \text{cats} \end{aligned}$$



And we have pluralization of event predicates in a domain of singular and plural events:

$$\begin{aligned} \llbracket \text{PURR} \rrbracket_{M,g} &= \{e_1, e_2, e_3\} && \text{purr} \\ \llbracket * \text{PURR} \rrbracket_{M,g} &= \{0, e_1, e_2, e_3, e_1 \sqcup e_2, e_1 \sqcup e_3, e_2 \sqcup e_3, e_1 \sqcup e_2 \sqcup e_3\} && \text{purr} \end{aligned}$$

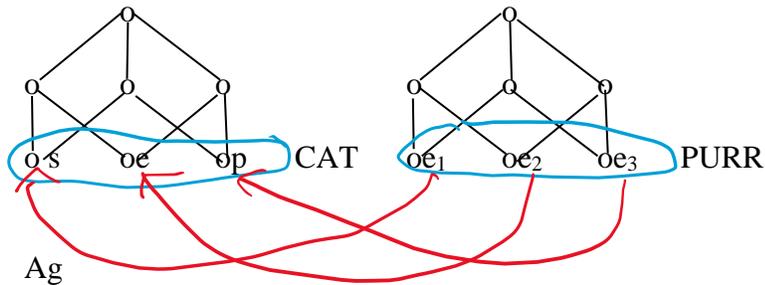


We have singular events and singular individuals, and thematic roles connecting these. We have plural events and plural individuals. We need plural roles connecting those. We lift these from the singular roles (Landman 2000).

$$\text{ATOM}_e = \{e_1 \in E: e_1 \sqsubseteq e \text{ and } e_1 \in \text{ATOM}_E\}$$

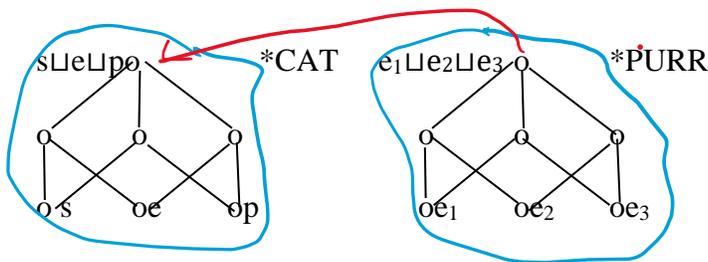
$$*Ag(e) = \begin{cases} \sqcup \{Ag(e_1) : e_1 \in ATOM_e\} & \text{if for every } e_1 \in ATOM_e \text{ } *Ag(e_1) \text{ is defined} \\ \perp & \text{otherwise} \end{cases}$$

Now suppose that Sasha, Emma and Pim are cats and Sasha purrs and Emma purrs and Pim purrs. We have three cats, and they all purr. The structures and roles are given as follows:



From this we can derive that:

$$s \sqcup e \sqcup p \in *CAT \quad e_1 \sqcup e_2 \sqcup e_3 \in *PURR \quad \text{and} \quad *Ag(e_1 \sqcup e_2 \sqcup e_3) = s \sqcup e \sqcup p$$



$$\text{So: } *CAT(s \sqcup e \sqcup p) \wedge *PURR(e_1 \sqcup e_2 \sqcup e_3) \wedge *Ag(e_1 \sqcup e_2 \sqcup e_3) = s \sqcup e \sqcup p$$

This means that:

$$\exists x[*CAT(x) \wedge |x|=3 \wedge \exists e[*PURR(e) \wedge *Ag(e)=x]]$$

There is a sum of three cats and there is a sum of purring events with that sum of cats as plural agent.

Make it past:

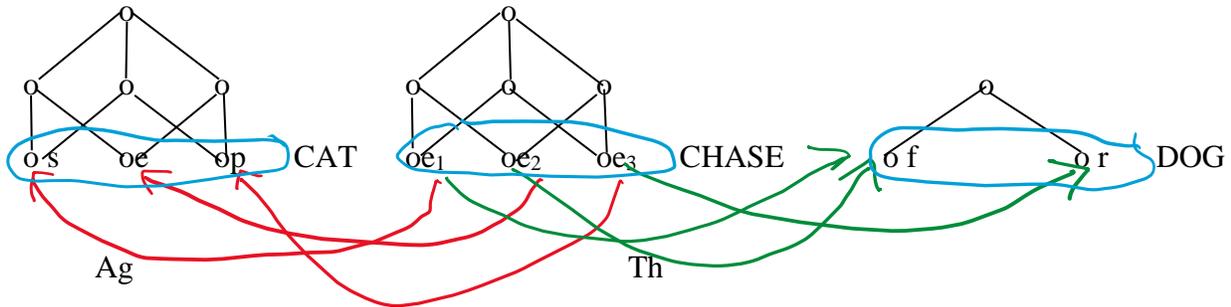
$$\exists x[*CAT(x) \wedge |x|=3 \wedge \exists e[*PURR(e) \wedge *Ag(e)=x \wedge \tau_w(e) < \text{now}]]$$

There is a sum of three cats and there is a sum of purring events located in w before now with that sum of cats as plural agent.

Three cats purred.

Distributive reading: There is a sum of three cats and each one of these three cats purred.

Now suppose that Sasha, Emma and Pim are cats and Fido and Rover are dogs, and Sasha chased Fido, Emma chased Fido as well, and Pim chased Rover. We have three cats, two dogs and three chasing events (we're ignoring the 0 event in the picture): and the roles are as given:



From this we can derive that:

$$s \sqcup e \sqcup p \in *CAT$$

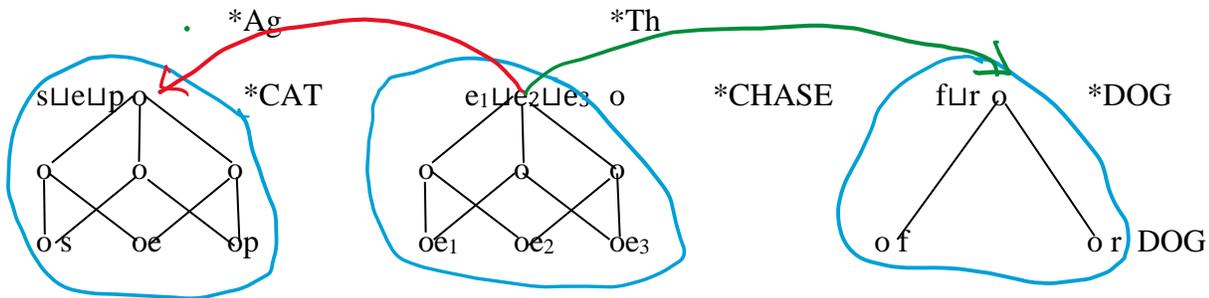
$$e_1 \sqcup e_2 \sqcup e_3 \in *CHASE$$

$$f \sqcup r \in *DOG$$

and

$$*Ag(e_1 \sqcup e_2 \sqcup e_3) = s \sqcup e \sqcup p$$

$$*Th(e_1 \sqcup e_2 \sqcup e_3) = f \sqcup r$$



Sasha and Emma and Pim are cats

Fido and Rover are dogs

$e_1 \sqcup e_2 \sqcup e_3$ is a sum of chasing events. Hence:

$$\exists e [*CHASE(e) \wedge *Ag(e) = s \sqcup e \sqcup p \wedge *Th(e) = f \sqcup r]$$

There is a sum of chasing events with Sasha and Emma and Pim as plural agent and Fido and Rover as plural theme.

Make it past:

$$\exists e [*CHASE(e) \wedge *Ag(e) = \wedge *Th(e) = f \sqcup r \wedge \tau_w(e) < \text{now}]$$

But that means that:

$$\exists x[*CAT(x) \wedge |x|=3 \wedge \exists y[*DOG(y) \wedge |y|=2 \wedge \exists e[*CHASE(e) \wedge *Ag(e)=x \wedge *Th(e)=y \wedge \tau_w(e)<now]]]]$$

There is a sum of three cats and there is a sum of two dogs and there is a sum of chasing events with that sum of cats as plural agent and that sum of dogs as plural theme, and that sum of chasing events is located in the past.

Three cats chased two dog

Cumulative reading: there is a sum of three cats and there is a sum of two dogs and every one of these cats chased one of these dogs and every one of these dogs was chased by one of these cat. The cumulative reading comes out at the basic plural reading.

Collective readings:

Landman 2000: Collective readings pattern with singular readings.

Landman 1989, 2000: Operation \uparrow maps sums onto group atoms. (Details in Landman 2000)

$$\exists x[*CAT(x) \wedge |x|=3 \wedge \exists y[*DOG(y) \wedge |y|=2 \wedge \exists e[CHASE(e) \wedge Ag(e)=\uparrow x \wedge Th(e)=\uparrow y \wedge \tau_w(e)<now]]]]$$

This expresses that $\uparrow(s\sqcup e\sqcup p)$, sasha and emme and pim as a group, chased $\uparrow(f\sqcup r)$ chased fido and rover as group: the first group chased the second. Nothing is expressed semantically as to who did what: *chase* does not semantically distribute to the individual cats and dogs.